Strategic design of public transport networks, frequencies and bus sizes

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Introduction
The basic single-line single-period model

Extension to two periods
A one-fleet system
A two-fleets system
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Introduction

The basic single-line single-period model
The basic model

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\[
VRC = VRC_O + VRC_U = B(c_B + c_K K) + Y_p v \bar{t}_v + Y_p w \bar{t}_w
\]

- \( B \) = fleet size, \( K \) = capacity of each vehicle, \( t_v, t_w \) average in-vehicle and waiting times
- \( c_B, c_K, p_w, p_v \) exogenous cost-related parameters
Introduction

The basic single-line single-period model

Solutions of the simple model

Everything can be expressed as a function of the frequencies

\[ VRC =Af + \frac{G}{f} + W, \text{ yielding:} \]

\[ f^* = \sqrt{\frac{Y}{TcB}} \left( \frac{p_w}{2} + tY \frac{l}{L}(p_v + c_K) \right), \quad K^* = \frac{Yl}{Lf^*} \]
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- Economies of scale: waiting time decreases, fleet per passenger decreases.
- Diseconomies of scale: Buses get larger, inducing longer times at stops.

Economies of scale prevail; all these effects get exhausted as $Y$ increases.
Relevance of the basic model

“By means of a simple bus line model it is possible to show that social cost minimisation results in a pattern of service characteristics which is radically different from most present services, mainly in these respects: given the demand, more buses should be running, and the buses should be much smaller.” (Jansson, 1980).
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Jansson (1984) tried to optimize the system under this scheme, but he could only do it by assuming equal frequencies across periods \( f_P = f_N \). Other authors have considered more than one period for other types of problems.
Posing the problem

We assume that $Y_i$, $T_i$ and $l_i$ depend on the period $(i = P, N)$. Each period lasts $E_i$, and we distinguish between capital and operational costs. Now we have to minimize
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$$+ \sum_{i=P,N} E_i Y_i (p_v t_{vi} + p_w t_{wi})$$

s.t. a) $K \geq \frac{Y_P l_P}{f_P L}$, b) $K \geq \frac{Y_N l_N}{f_N L}$
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Everything is written as a function of $f_P$, $f_N$. At least one restriction must be active. $\max(B_P, B_N) = B_P$. 
Size given by the peak

Let us first analyze the case in which the peak restriction is active but the off-peak one is not: \( K = \frac{Y_P}{f_P L} \). This yields

\[
VRC_2 = A_P f_P + A_N f_N + \frac{G_P}{f_P} + \frac{G_N}{f_N} + \delta \frac{f_N}{f_P} + U
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F.O.C. \(\Rightarrow\) equations of degree 5. But:

\[
f_P^* = \sqrt{\frac{G_P + \delta f_N^*}{A_P}}, f_N^* = \sqrt{\frac{G_N}{A_P + \delta / f_P^*}}
\]

\((f_1^* = \sqrt{\frac{G}{A}})\)
Analytical results

- $f_P^*$ is larger than in the isolated case, because we want smaller buses as they also run at the off-peak.
- $f_N^*$ might be larger or smaller than in the isolated case: no capital costs ($\uparrow$), but larger buses and frequency does not impact their size ($\downarrow$).
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Crossed effects can also be proved:
- $\frac{\partial f^*_P}{\partial Y_N} > 0$, because vehicles’ size becomes more important.
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- Then $\frac{Y_P l_p}{f_P^* L} = K = \frac{Y_N l_N}{f_N^* L} \Rightarrow$ everything can be expressed as a function of $f_N^*$ and explicit solutions can be found

$$f_N = \sqrt{\frac{t Y_P c_{KC} Y_N l_N}{T_N E_N c_{BO} + T_P c_{BC} Y_P l_P / (Y_N l_N) + T_P E_P c_{BO} Y_P l_P / (Y_N l_N) + E_P Y_P + E_N Y_N} + \frac{p_w Y_N (E_N + E_P l_P / l_N)}{2}}$$
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- Analytical results depend on the value of the parameters.
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Extension to two periods

A two-fleets system
The general idea

Which is the best way to design for a two-periods scheme? A reasonable strategy is allowing for two fleets of different size:

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- Both running at both periods.
- Each fleet runs at only one period.
- Small buses running alone at the off-peak, large buses complementing at the peak.

Problem: at the peak, different buses will have different time at stops. We will use a holding strategy.
The holding strategy

\[ H = (K_L - K_S) \frac{L}{l_P} \cdot t \]

The equations that determine the system are found, and the system can be optimized (numerically).
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- Extension to two periods
- Comparison of these systems

Extension to two periods

Comparison of these systems
They have results that are too similar. Let us look into these results in more depth.
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Peak: off-peak users are favored by the one-fleet system, off-peak users by the two-fleets system.
Extension to a network

Comparing basic lines structures
Complexity of this extension

The extension towards a network is way more complex:

\[ VRC = X \sum_{L} \left( c_B + c_K \right) + Y \left( p_v \bar{v} t_v + p_w \bar{w} t_w + p_a \bar{a} t_a + p_T \bar{T} T \right) \]
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- We need to find the optimal *lines structure*, i.e., the set of routes of the transit lines. Optimizing the lines structure, misconsidering frequencies, is already an NP-Hard problem.
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- Passengers can **choose routes**. These routes depend on the frequencies, which in turn need to be adjusted to carry every passenger.
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- Passengers can choose routes. These routes depend on the frequencies, which in turn need to be adjusted to carry every passenger.

We are not solving the problem, but understanding it better. Fleet and bus capacity are now defined per line, and two spatial elements appear in the users’ costs: access time and number of transfers.

\[
VRC = \sum_{L} B_L(c_B + c_K K_L) + Y(p_v \bar{t}_v + p_w \bar{t}_w + p_a \bar{t}_a + p_T \bar{T})
\]
The parametric city

We analyze several aspects of the problem over this city model

SC=Subcenter, P=Periphery.
Role of the parameters

\[ \alpha \rightarrow 1 \text{ Monocentric,} \quad \beta \rightarrow 1 \text{ Polycentric,} \quad \gamma \rightarrow 1 \text{ Dispersed} \]

\[ \alpha + \beta + \gamma = 1 \]
Basic lines structures

Four lines structures over this city model.

Feeder – Trunk (FT)  
Hub & Spoke (HS)  
No transfers (NT)  
No stops (NS)
Results of the basic structures

For each lines structure, and for each $\alpha, \beta, \gamma, Y$:

1. $VRC$ is expressed as a function of the vector of frequencies.
2. The frequencies are optimized.
3. The resulting minimum $VRC$ is obtained.
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We can identify the best lines structure for each OD pattern:

\[ \alpha - Y \text{ space } (\beta = \gamma) \]
\[ \alpha - \beta \text{ space } (Y = 24000) \]
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- Extension to a network
- Scale economies and directness

Extension to a network

Scale economies and directness
The concept of directness

\[ DSE = \frac{\text{Average costs}}{\text{Marginal costs}} \]

There are scale economies iff \( DSE > 1 \). \( DSE \) increases each time a lines structure changes... why?
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- Passengers follow shorter routes.
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This suggests the definition of the Directness of a lines structure, encompassing these three characteristics.
Directness over the parametric city

<table>
<thead>
<tr>
<th>Structure</th>
<th>FT</th>
<th>HS</th>
<th>NT</th>
<th>NS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfers per trip</td>
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These are indices before assigning passengers. Directness increases:
FT → HS → NT → NS
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**DSE in the parametric city**

\((\alpha = 0.5, \beta = \gamma = 0.25)\)

Best structure: HS → NT → NS: Directness increases!.
**DSE in the parametric city**

\[ \alpha = 0.5, \beta = \gamma = 0.25 \]

Best structure: HS → NT → NS: Directness increases!

**DSE curve:** always larger than 1, it decreases to 1, jumping at each change in lines structure.
Evolution of the indices

Indices decrease (grossly speaking). There are trade-offs between the components of directness. In addition, each time the lines structure changes:

- Waiting time increases (diseconomies of scale).
- The number of seats decreases (idle capacity is reduced; economies of scale).
Conclusions

1. Considering temporal and spatial heterogeneity adds complexity to public transport models. Relevant qualitative effects can be identified.

2. Different strategies are possible to face the two-periods problem. In this research, we analyzed and compared two strategies: one fleet or two fleets.

3. In the one-fleet system, buses always run full at the peak. Vehicles’ capacity lies in between the ones obtained by optimizing each period independently.

4. In the two-fleets system, a holding strategy is needed.

5. Both strategies yield similar total costs. The two-fleets system is better for off-peak users and worse for peak users.
Conclusions

6. Spatial exact optimization cannot be achieved; alternative approaches are needed.

7. We identified the best basic lines structure for different city and demand patterns.

8. We identified a novel source of economies of scale: the level of directness of the lines structure. As the number of passengers increases, routes become shorter, with less stops and less transfers. Scale economies get eventually exhausted.

9. *Spatial density is another source of scale economies.* Total waiting costs = total walking costs. *Increasing spatial density reduces the directness.*
References

Thank you for your attention!