Strategic design of public transport networks, frequencies and bus sizes

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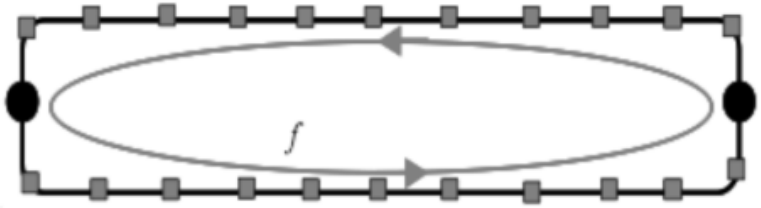
Introduction

The basic single-line single-period model

The basic model

In the classic single-line single-period model, Y passengers are distributed accross L which is completed in T, they all travel l and take t to board/alight.



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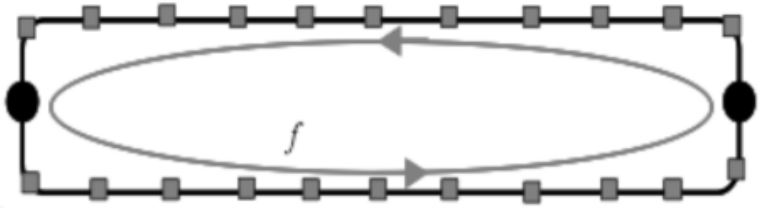

$$VRC = VRC_O + VRC_U = B(c_B + c_K K) + Yp_v\bar{t_v} + Yp_w\bar{t_w}$$

- B=fleet size, K=capacity of each vehicle, $\bar{t_v}, \bar{t_w}$ average in-vehicle and waiting times
- c_B, c_K, p_w, p_v exogenous cost-related parameters

Solutions of the simple model

Everything can be expressed as a function of the frequencies $VRC = Af + G/f + W$, yielding:

$$f^* = \sqrt{\frac{Y}{Tc_B}\left(\frac{p_w}{2} + tY\frac{l}{L}(p_v + c_K)\right)}, K^* = \frac{Yl}{Lf^*}$$

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- Economies of scale: waiting time decreases, fleet per passenger decreases.
- Diseconomies of scale: Buses get larger, inducing longer times at stops.

Economies of scale prevail; all these effects get exhausted as Y increases.

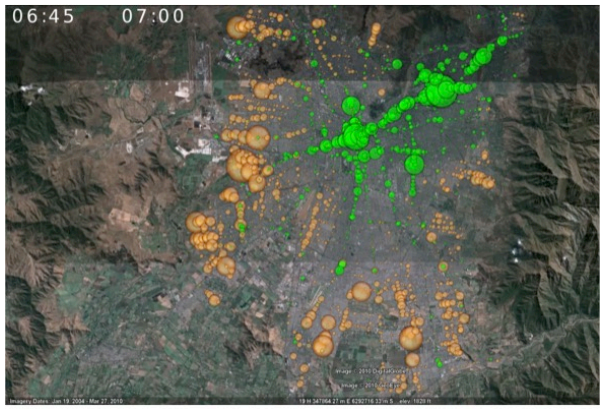
Relevance of the basic model

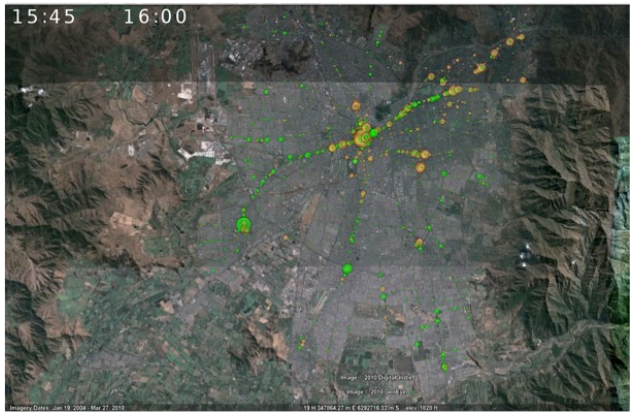
"By means of a simple bus line model it is possible to show that social cost minimisation results in a pattern of service characteristics which is **radically different from most present services**, mainly in these respects: given the demand, **more buses should be running, and the buses should be much smaller**." (Jansson, 1980).

Extension to two periods

A one-fleet system

Motivation





Source: Munizaga and Palma (2012)

Jansson (1984) tried to optimize the system under this scheme, but he could only do it by assuming equal frequencies accross periods $f_P = f_N$. Other authors have considered more than one period for other types of problems.

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We assume that Y_i, T_i and l_i depend on the period ($i = P, N$). Each period lasts E_i, and we distinguish between capital and operational costs. Now we have to minimize

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$$VRC_2 = \text{máx}(B_P, B_N)(c_{BC} + Kc_{KC}) + (B_P E_P + B_N E_N)(c_{BO} + Kc_{KO})$$

$$+ \sum_{i=P,N} E_i Y_i (p_v \bar{t_{vi}} + p_w \bar{t_{wi}})$$

$$s.t. \;\; \text{a)} K \geq \frac{Y_P l_P}{f_P L}, \text{b)} K \geq \frac{Y_N l_N}{f_N L}$$

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Everything is written as a function of f_P, f_N. At least one restriction must be active. $\text{máx}(B_P, B_N) = B_P$.

Size given by the peak

Let us first analyze the case in which the peak restriction is active but the off-peak one is not: $K = \frac{Y_P l_P}{f_P L}$. This yields

$$VRC_2 = A_P f_P + A_N f_N + \frac{G_P}{f_P} + \frac{G_N}{f_N} + \delta \frac{f_N}{f_P} + U$$

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F.O.C. $\Rightarrow$ equations of degree 5. But:

$$f_P^* = \sqrt{\frac{G_P + \delta f_N^*}{A_P}}, f_N^* = \sqrt{\frac{G_N}{A_P + \delta / f_P^*}}$$

$$\left(f_1^* = \sqrt{\frac{G}{A}} \right)$$

Analytical results

- f_P^* is larger than in the isolated case, because we want smaller buses as they also run at the off-peak.
- f_N^* might be larger or smaller than in the isolated case: no capital costs ($\uparrow$), but larger buses and frequency does not impact their size ($\downarrow$).

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A two-fleets system

The general idea

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- Each fleet runs at only one period.
- ***Small* buses running alone at the off-peak, *large* buses complementing at the peak**.

Problem: at the peak, different buses will have different time at stops. We will use a *holding* strategy.

The holding strategy

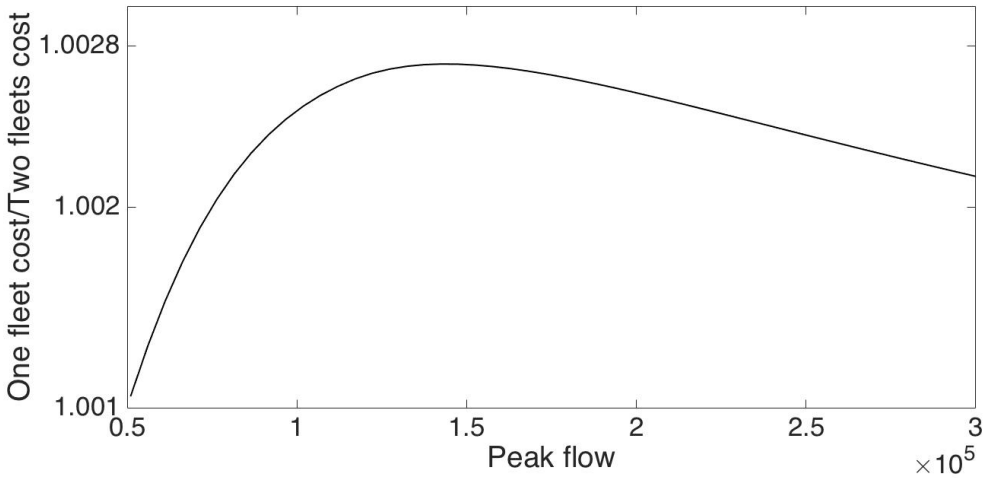
$$H = (K_L - K_S)\frac{L}{l_P} \cdot t$$

The equations that determine the system are found, and the system can be optimized (numerically).

Extension to two periods

Comparison of these systems

General comparison



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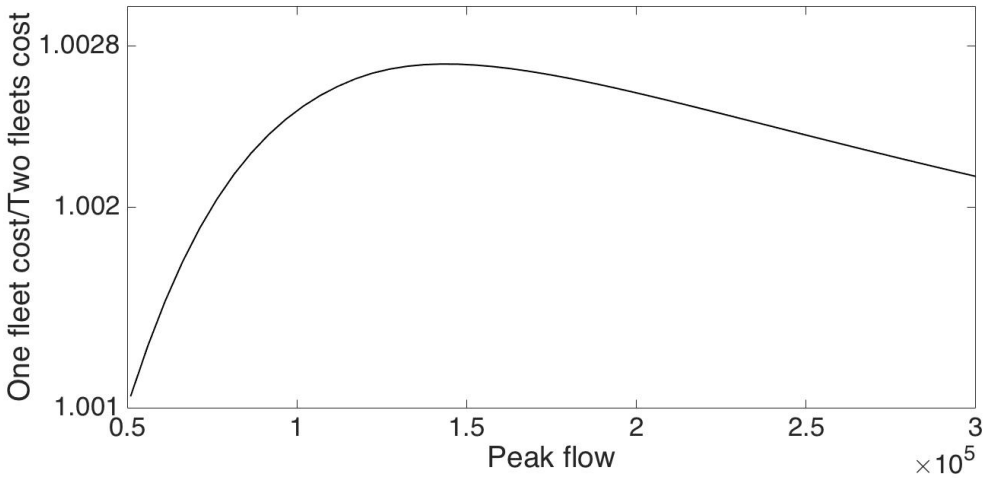

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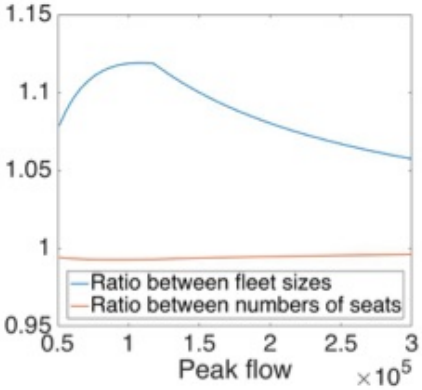
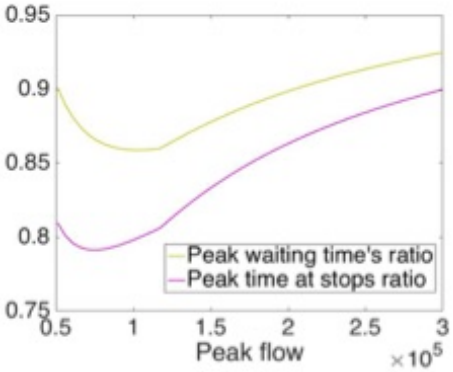
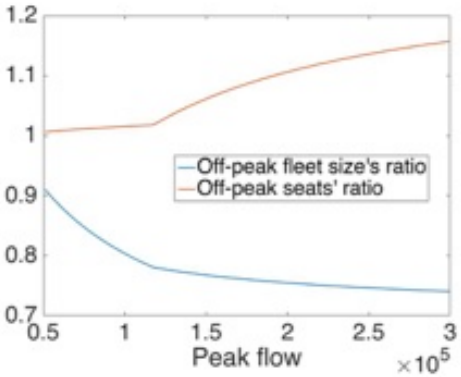
Results per period

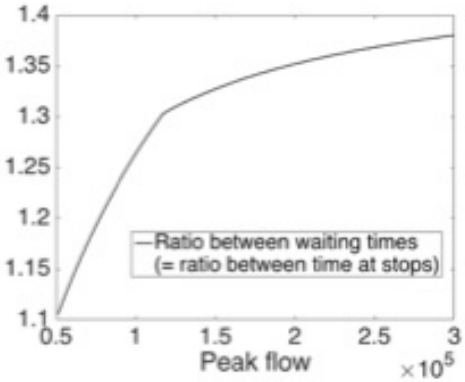
Peak:





Off-peak:





Peak users are favored by the one-fleet system, off-peak users by the two-fleets system.

Extension to a network

Comparing basic lines structures

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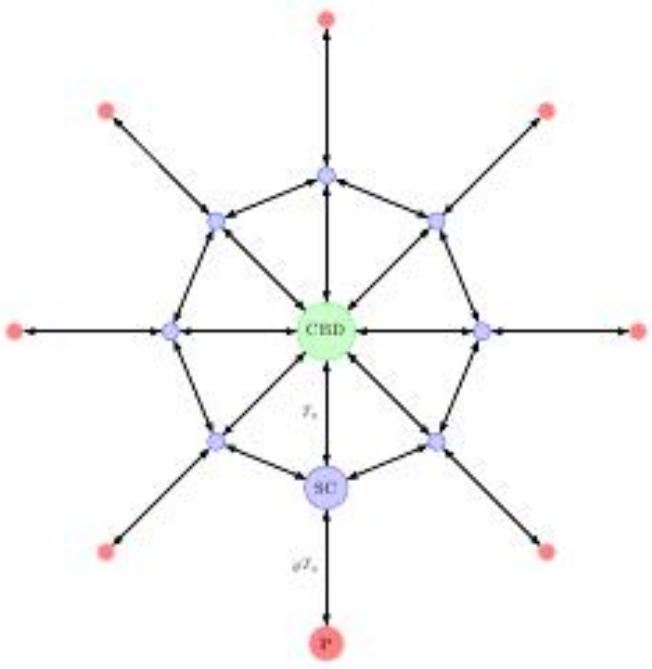
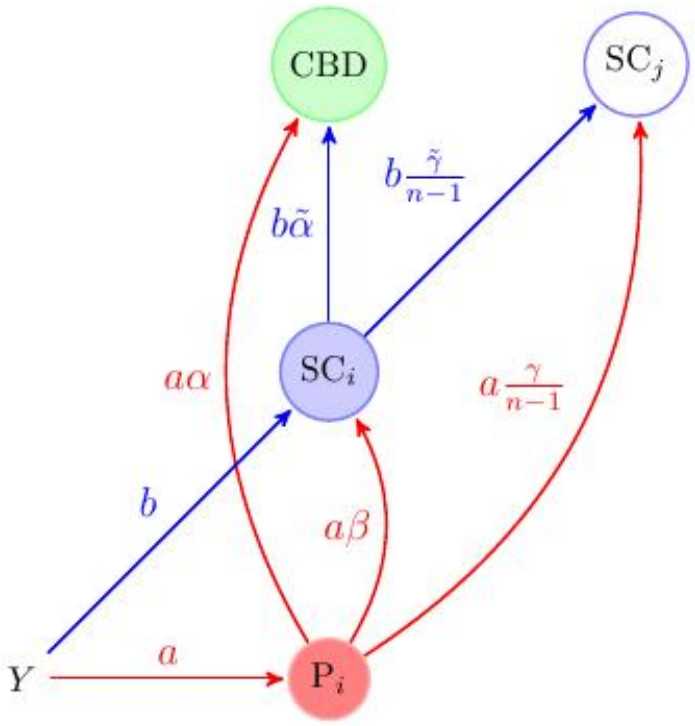
We are not solving the problem, but understanding it better. Fleet and bus capacity are now defined per line, and two spatial elements appear in the users' costs: access time and number of transfers.

$$VRC = \sum_L B_L(c_B + c_K K_L) + Y(p_v \bar{t_v} + p_w \bar{t_w} + p_a \bar{t_a} + p_T \bar{T})$$

The parametric city

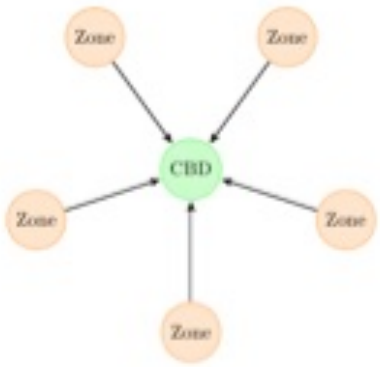
We analyze several aspects of the problem over this city model

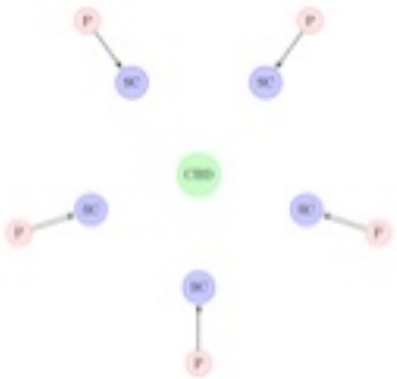


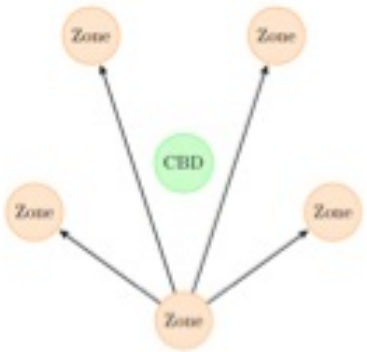


SC=Subcenter, P=Periphery.

Role of the parameters





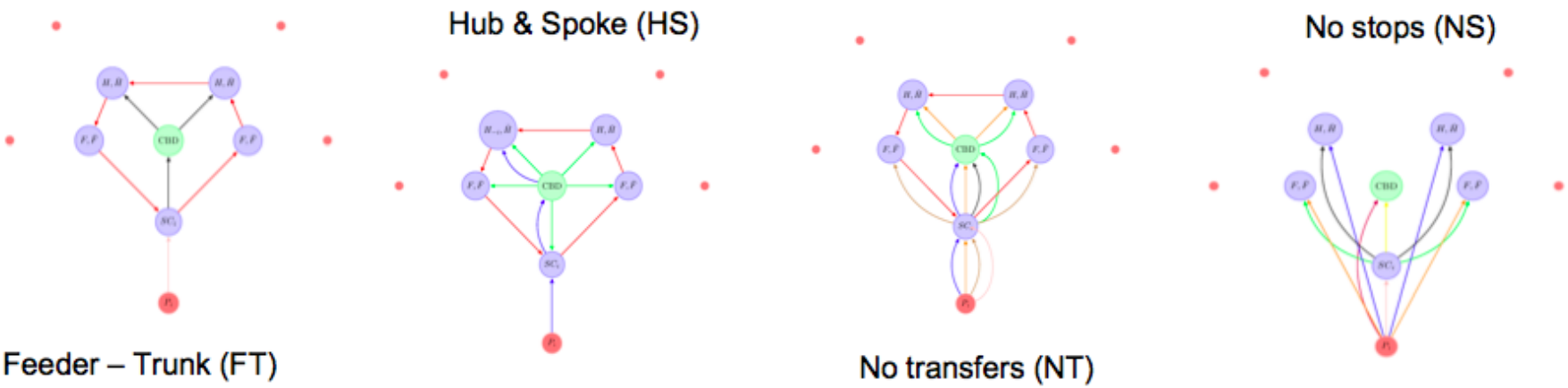


$\alpha \to 1$ Monocentric, $\beta \to 1$ Polycentric, $\gamma \to 1$ Dispersed

$$\alpha + \beta + \gamma = 1$$

Basic lines structures

Four lines structures over this city model.



Results of the basic structures

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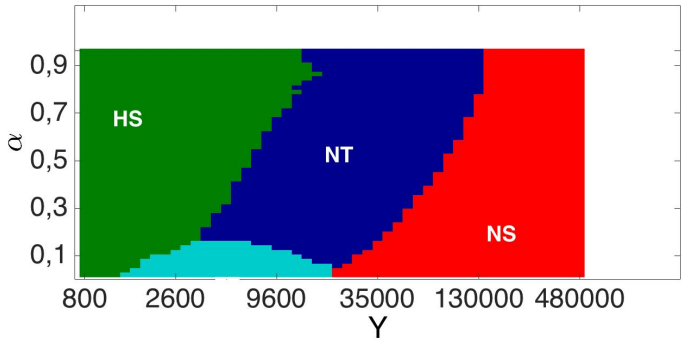
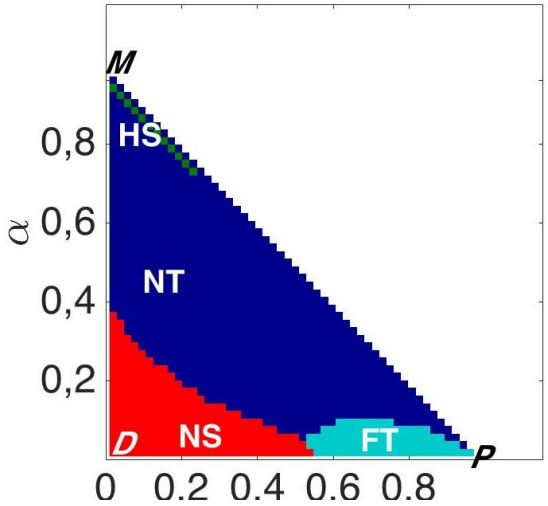
We can identify the *best* lines structure for each OD pattern:



$\alpha - Y$ space ($\beta = \gamma$)



$\alpha - \beta$ space ($Y = 24000$)

Extension to a network

Scale economies and directness

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- The number of stops decreases.

This suggests the definition of the *Directness* of a lines structure, encompassing these three characteristics.

Directness over the parametric city

Structure	FT	HS	NT	NS
Transfers per trip	0.47	0.35	0	0
Stops per trip	3.06	3.06	3.06	2
Distance traveled/Min distance	1	1	1	1

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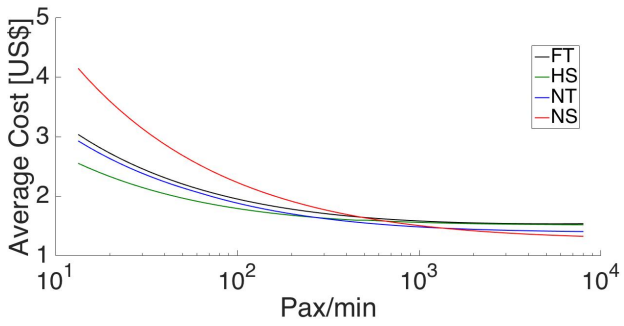
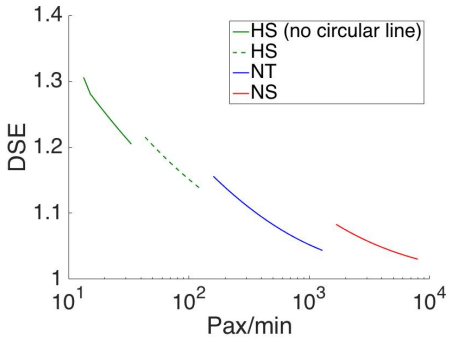
These are indices before assigning passengers. Directness increases: FT $\rightarrow$ HS $\rightarrow$ NT $\rightarrow$ NS

DSE in the parametric city

$(\alpha = 0{,}5, \beta = \gamma = 0{,}25)$





Best structure: HS→NT→NS: Directness increases!.

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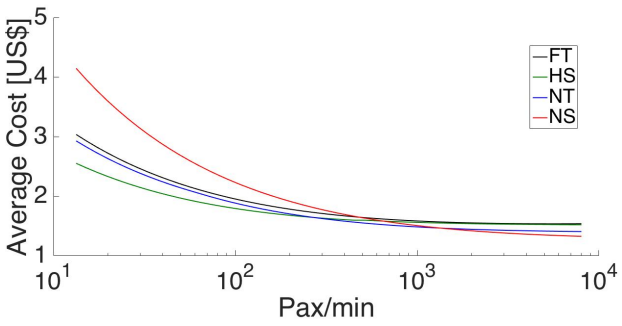

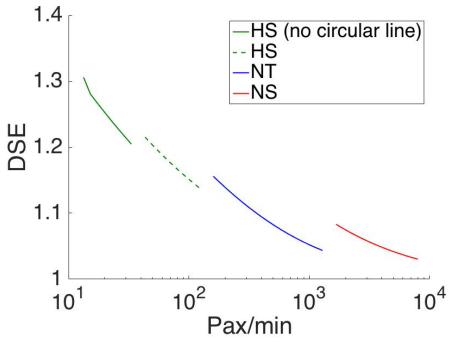

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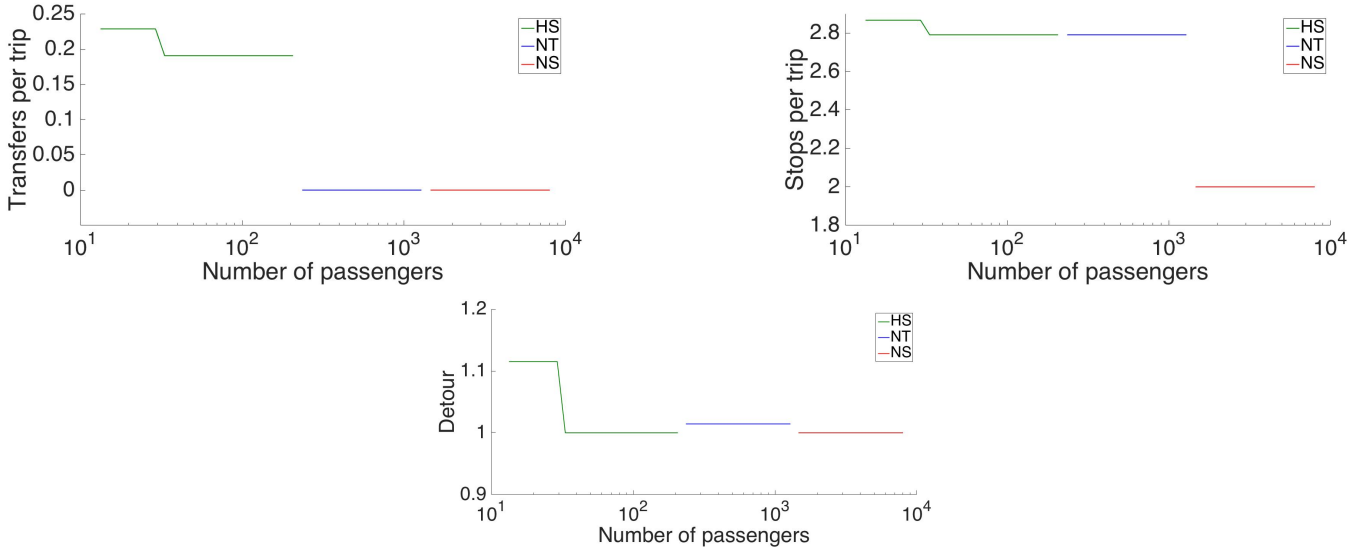
DSE curve: always larger than 1, it decreases to 1, jumping at each change in lines structure.

Evolution of the indices



Indices decrease (grossly speaking). There are trade-offs between the components of directness. In addition, each time the lines structure changes:

- Waiting time increases (diseconomies of scale).
- The number of seats decreases (idle capacity is reduced; economies of scale).

Conclusions

1. Considering temporal and spatial heterogeneity adds complexity to public transport models. Relevant qualitative effects can be identified.
2. Different strategies are possible to face the two-periods problem. In this research, we analyzed and compared two strategies: one fleet or two fleets.
3. In the one-fleet system, buses always run full at the peak. Vehicles' capacity lies in between the ones obtained by optimizing each period independently.
4. In the two-fleets system, a holding strategy is needed.
5. Both strategies yield similar total costs. The two-fleets system is better for off-peak users and worse for peak users.

Conclusions

6. Spatial exact optimization cannot be achieved; alternative approaches are needed.
7. We identified the best basic lines structure for different city and demand patterns.
8. We identified a novel source of economies of scale: the level of directness of the lines structure. As the number of passengers increases, routes become shorter, with less stops and less transfers. Scale economies get eventually exhausted.
9. *Spatial density is another source of scale economies. Total waiting costs = total walking costs. Increasing spatial density reduces the directness.*

References

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Thank you for your attention!